A Derivatives Splitting Approach to Sensitivity Analysis of Magnets Design

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The paper proposes a mixed analytical-numerical approach for an effective evaluation of the variation of the magnetic field generated by complex coils due to deformations with respect to the nominal configuration. The coils are described with a set of synthetic parameters and the deformations can be described as variations of such parameters. The work is focused on the context of sensitivity analysis but the methodology is quite general and can be easily extended in other fields.

Index Terms—**Coils, Tolerance Analysis, High-Performance Computing.**

I. Introduction

Due to their complexity and required accuracy, the design of magnets used in a number of applications such as nuclear resonance imaging, controlled thermonuclear fusion or for particle accelerators is very demanding. As a matter of fact, together with the magnetic field evaluation accuracy, an additional priority is the robustness to prevent the degradation of the performance due to the inevitable discrepancies from the nominal design.

In order to assess the effect of such variations, it is useful to perform a tolerance analysis. For this purpose, the coil shape can be described with a limited number of geometrical parameters $\underline{p}$ and the effect of the variations is evaluated as the variation of a performance function $\mathcal{F}(\ \boldsymbol{B}(\underline{p};\ \boldsymbol{r})\ )$, where $\mathbf{B}$ is the magnetic field and $\boldsymbol{r}$ is the field point. Typical examples of the performance function are the field uniformity in NMR magnets [1] or the error field harmonics in fusion devices [2].

The analysis can take benefit from the calculation of the gradient of $\mathcal{F}$ on the $\underline{p}$ components (Sensitivity Array, SA) [3]. Of course, such evaluation requires the preliminary evaluation of the derivatives of the magnetic field $\mathbf{B}$ itself with respect to $\underline{p}$. Unfortunately, the evaluation of the magnetic field generated by complex coils can be rather expensive.

An effective approach to reach good accuracy is the discretization of the coil in a number of elementary sources as current segments, sheets or bricks. The subsequent evaluation of the magnetic field, under the assumption of absence of ferromagnetic materials, can be performed as the superposition of the contributes to the field of each of the elementary sources. The availability of Green functions in analytical form can be very useful to reduce the required computational burden, especially when high-performance architectures are available [4].

This paper proposes an effective approach to evaluate the magnetic field derivatives. The method takes advantage of the chain rule to split the dependence of the field on $\underline{p}$ on two components [5]: (i) a geometrical factor representing the impact of $\underline{p}$ on the actual magnet shape and (ii) a magnetic factor coming from the Green function describing how the field depends on the magnet shape. The splitting approach can be very effective when the sensitivity with respect to a large set of possible variations of the parameters is required. Due to lack of room, here just the mathematical formulation and a simple example are reported; more details will be provided in the final version of the paper.

II. Mathematical Formulation

Due to its complexity, in practical applications a magnet is discretized in N_s elementary sources suitably connected. The geometry information of the k-th source can be collected in an array $\underline{q_k}$. Of course, if the shape of the magnet is assumed depending on a set of parameters $\underline{p}$, then $\underline{q_k}$ is a function of $\underline{p}$ as well. Therefore, the magnetic field in $\boldsymbol{r}$ can be written as:

$$\boldsymbol{B}\left(\underline{p};\ \boldsymbol{r}\right) = \sum_{k=1}^{N_s} \boldsymbol{B_k}\left(\underline{q_k}\left(\underline{p}\right);\ \boldsymbol{r}\ ,\boldsymbol{J_k}\right) \tag{1}$$

where $\boldsymbol{B_k}$ is the magnetic field generated by the k-th source with current density $\boldsymbol{J_k}$.

The derivative of the magnetic field respect the i-th parameter of $\underline{p}$ can be evaluated by using the chain rule

$$\frac{\partial \boldsymbol{B}}{\partial p_i} = \sum_{k=1}^{N_s} \frac{\partial \boldsymbol{B_k}}{\partial p_i} = \sum_{k=1}^{N_s} \sum_{j=1}^{N_p(k)} \frac{\partial \boldsymbol{B_k}}{\partial q_{k_j}} \frac{\partial q_{k_j}}{\partial p_i} \tag{2}$$

where $N_p(k)$ is the number of parameters describing the k-th source.

Eq. (2) shows how the sensitivity of the field on the geometrical parameters $\underline{p}$ can be split in two independent components. The magnetic factor does not depend on the shape parameters value; therefore, when just the impact of alternative shapes should be analyzed, just the second factor needs to be updated.

The method is quite useful in a wide range of applications when the variation of the magnetic field due to small variations $\Delta\underline{p}$ (due, for example, to mechanical deformations) with respect to the nominal ones $\underline{p_0}$ is required. In this case, a first order Taylor expansion can be used to recover the variation of the field by using the sensitivity (2).

III. Elementary Sources

A number of elementary models useful to describe magnetic sources have been proposed in literature. In this paper, the attention is focused on three models (see Fig. 1): *(a) Current Segment* [6]*:* the vector of parameters $\underline{q}$ includes the set of coordinates of the endpoints couples; *(b) Planar Sheet* [7]-[8]*:* $\underline{q}$ is assumed to contain the sets of coordinates of four vertices; *(c) Current 3D bricks* [9]: $\underline{q}$ includes the coordinates of the set of the 8 vertex of the uniform current brick.

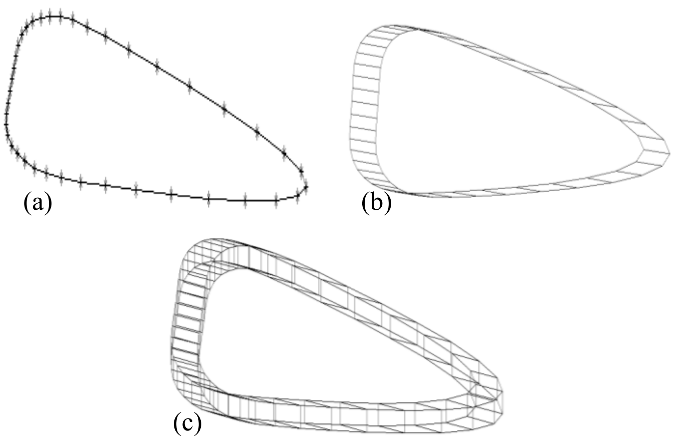

Fig. 1. A complex 3D coil discretized (a) with current segments, (b) with sheets and (c) with bricks.

IV. Example Of Application

Here a quite simple application is discussed with the aim of showing the effectiveness and the limits of the procedure.

A massive magnet designed to generate the poloidal field in Toroidal fusion devices [10] is here considered (See. Table I). The magnet is composed by a circular winding of about one hundred of turns of superconductor cable. A parametric analysis on the impact of elliptical deformations has been considered here. A strong advantage comes from the splitting because just the geometrical factor needs to be updated providing this manner a computing burden reduction.

TABLE I
Configuration Of The Example

PF Coil Radius	12 m
PF Planar Plane	z = 3.275 m
Coil Current	10 MA
Field point D_1 coordinates	{x = 9.32; y = 0; z = 2.36 } m
Field point D_2 coordinates	{x = 8.81; y = 0; z = -2.33} m
Field point D_3 coordinates	{x = 3.81; y = 0; z = -5.12} m

In Fig. 2 the variation $\Delta\boldsymbol{B}$ on three field points due to the deformations is shown normalized on the flux density $\boldsymbol{B_0}$ of the nominal configuration. The variation of the field ($\Delta\boldsymbol{B_T}$) is evaluated both with a Taylor polynomial of first order and with the analytical current segment model ($\Delta\boldsymbol{B_S}$). The quality of the Taylor approximation is shown in Fig. 3 where the quantity $|\Delta\boldsymbol{B_T} - \Delta\boldsymbol{B_S}|/|\boldsymbol{B_0}|$ is compared with the coil ellipticity, defined as ratio between major and minor axis.

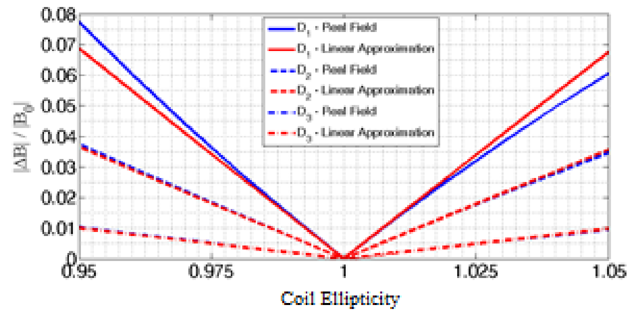

Fig: 2. Effect of the deformations on the magnetic field.

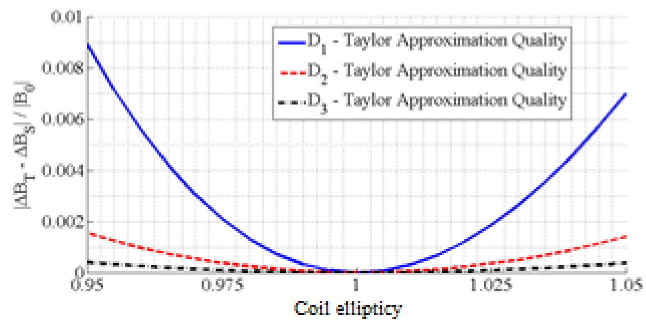

Fig: 3. Quality of the Taylor polynomial of first order.

The results of the analyzed case show that the accuracy of the linear estimation has better values for small deformations of the sources and large distances.

This result can be a strong advantage when a wide range of deformations need to be considered as in sensitivity analysis: in fact, computations of the magnetic field can be expensive while by using the linear approximation it can be roughly reduced to simple multiplications.

V. References

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